

## BASICS OF SIGNAL ENCODING



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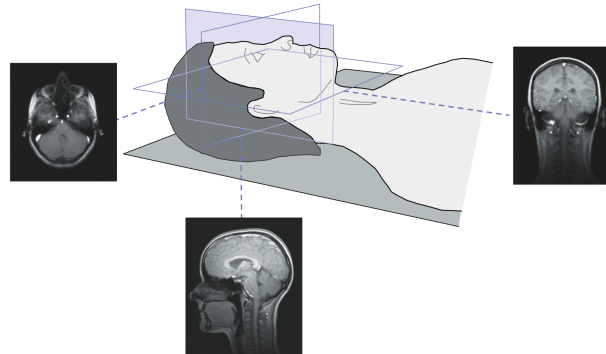
### Learning Outcomes

- **Magnetic field gradients** form the basis of MR signal localization
- 2D slices are produced by the combination of an **excitation RF pulse and simultaneous slice-select gradient**
- The in-plane MR signal is encoded in terms of the spatial frequencies of the object using **phase-encoding and frequency-encoding gradients**
- We sample every spatial frequency that exists within the image and then **Fourier transform** these data (known as k-space) to produce an image
- Inadequate or erroneous k-space sampling leads to certain **artifacts**



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## Magnetic Resonance Imaging or Tomography

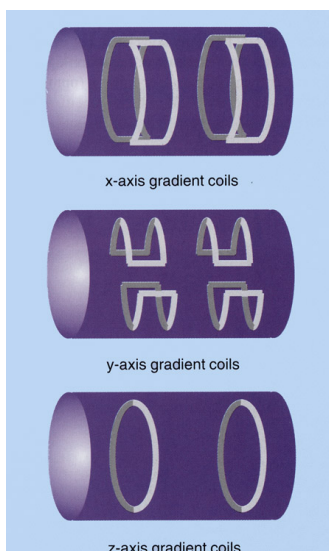


- Tomography:
    - Tomos: cut, section
    - Graphein: to write
- Signal measured as a function of space

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## Imaging Gradients

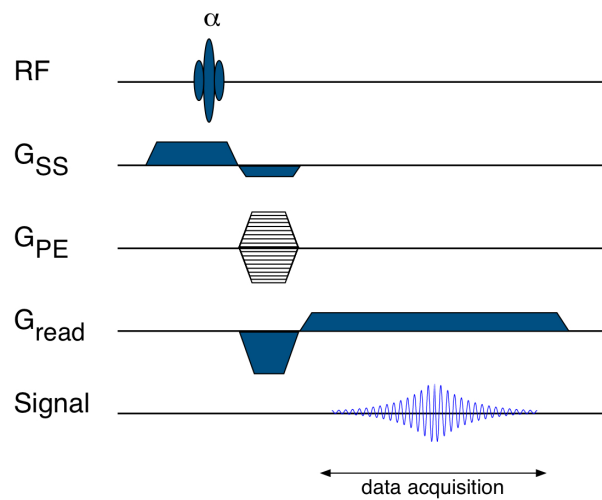


- There are coils which produce the main magnetic field ( $B_0 = 7\text{T}$  in our case).
- In addition, there are several other coils making small, additional magnetic fields which vary linearly in space. These are called **gradient coils**.
- These small additional magnetic fields are present in X, Y and Z direction. Without them, we can not say where in the tissue the signals come from.

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## Basic Pulse Sequence



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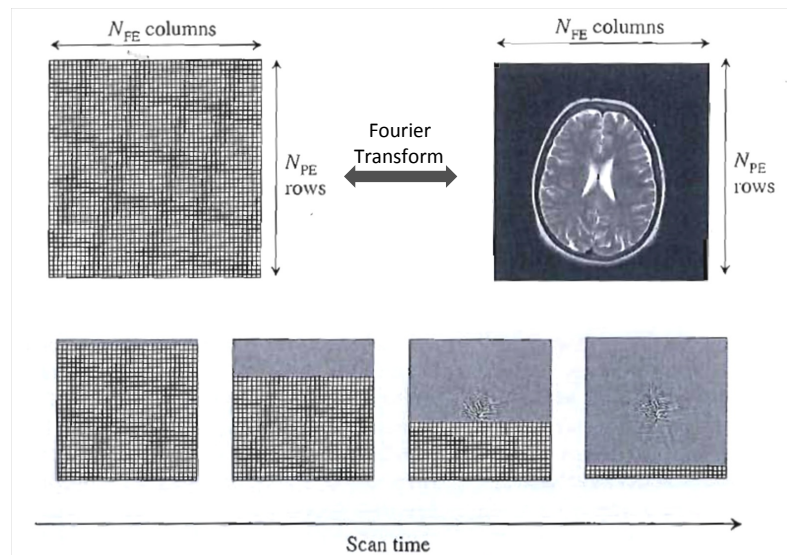
## Slice, Phase and Frequency Encoding in K-Space

- **Slice-encoding:** a combination of an RF pulse and a gradient excites spins within a plane  $\rightarrow$  slice-selective gradient
- **Phase-encoding:** a constant gradient is applied for a specific duration before acquisition of the signal for every phase-encoding step
- **Frequency-encoding:** a constant readout gradient is applied during acquisition of the signal (echo)
- Total scan time:  $NSA \times N_{PE} \times TR$

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## From K-Space to Image Space



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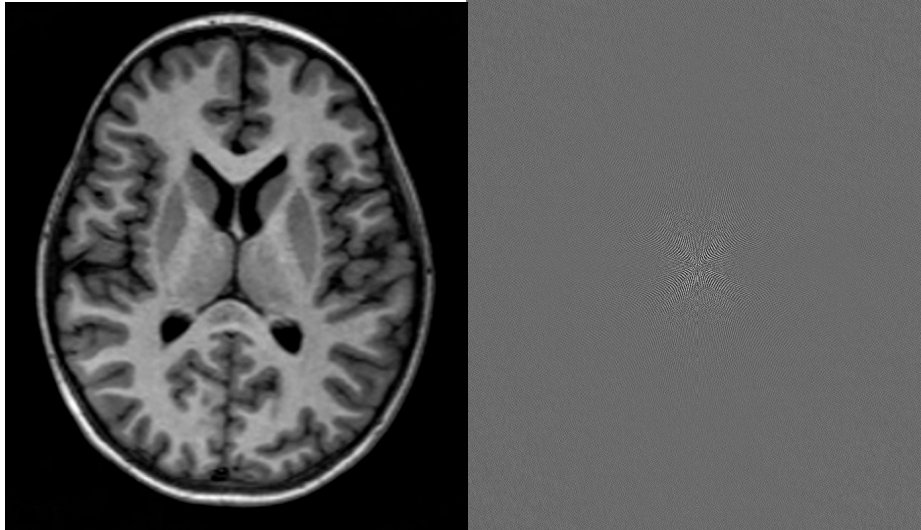
## The K-Space

- Think of it as temporary image space
- Data is collected in k-space and then FT to image space
- The matrix size of the final image is the same as the matrix size of k-space
- There is no pixel-to-pixel correspondence between k-space and image space; instead, each pixel of image space contains information from the entire k-space, and vice-versa
- Data in the middle of k-space contain SNR and contrast information
- Data from the edges of k-space contain all the information about the image resolution

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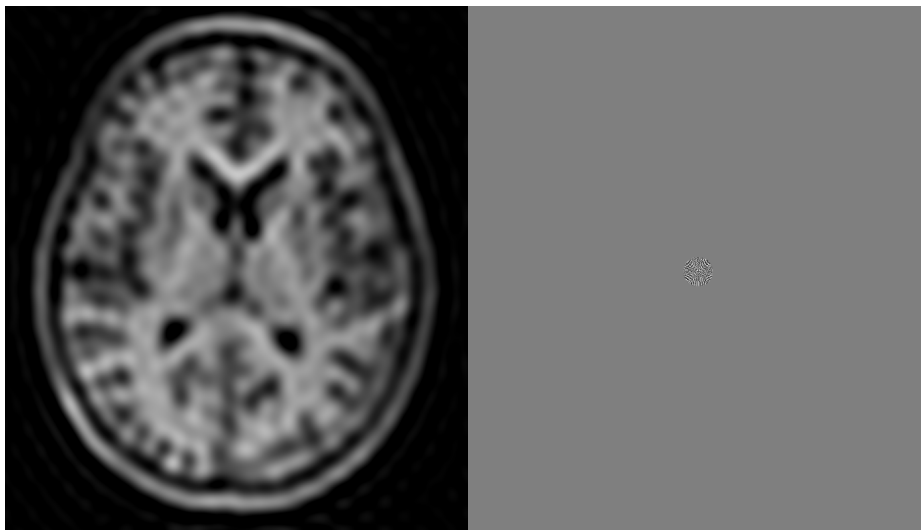


### Understanding K-Space: Image and K-space



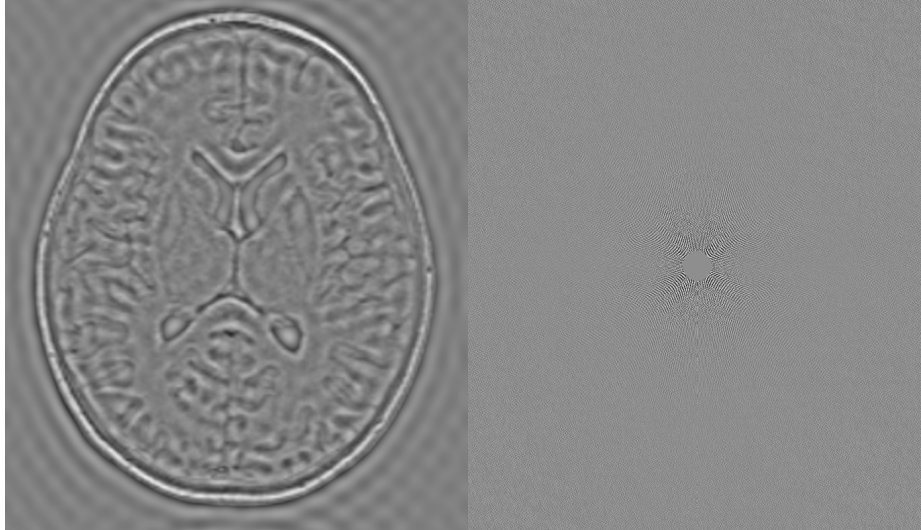
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### Understanding K-Space: Low-Pass Filtering



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## Understanding K-Space: High-Pass Filtering



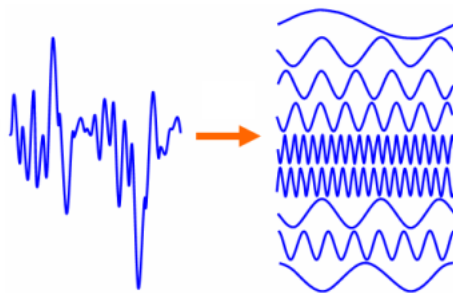
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## How to move between image and k-space: Inverse Fourier Transform



Joseph Fourier



Any signal in time could be split into a series of Fourier components, each at a different frequency

*Time domain* → FT → *Frequency Domain*

*K-space domain* → IFT → *Image domain*

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## Time domain, Frequency domain, K-space and Image domain: How it all relates?

Time Domain  $\rightarrow$  FT  $\rightarrow$  Temporal Frequency Domain

$t \rightarrow$  FT  $\rightarrow f$

seconds  $\rightarrow$  FT  $\rightarrow$  Hz

K-space domain or  
Spatial Frequency Domain  $\rightarrow$  IFT  $\rightarrow$  Image Domain

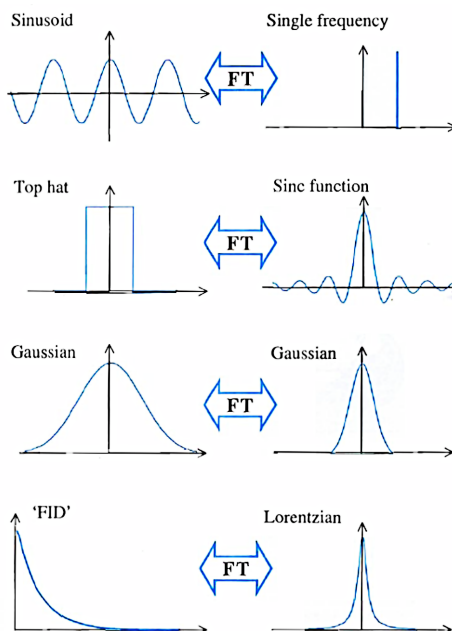
$k \rightarrow$  IFT  $\rightarrow r$

1/cm  $\rightarrow$  IFT  $\rightarrow$  cm



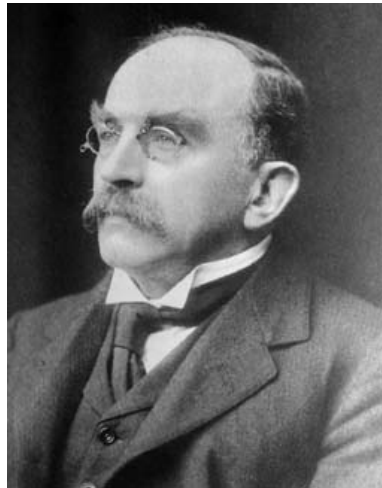
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## Fourier Pairs



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## Back to Image Encoding: Larmor Frequency



Sir Joseph Larmor

$$f = \frac{\gamma}{2\pi} B_0 \text{ or } \omega = \gamma B_0$$

$$\frac{\gamma}{2\pi} = 42 \text{ MHz T}^{-1} \quad \gamma = 2.67 \times 10^8 \text{ radians s}^{-1} \text{ T}^{-1}$$

Spin precession is proportional  
to magnetic field strength

Gradients

Signal localization

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## Magnetic Field Gradients

- Gradient = spatially-linear variation in the static field strength in the z-direction

$$G_x = \frac{\partial B_z}{\partial x} \quad G_y = \frac{\partial B_z}{\partial y} \quad G_z = \frac{\partial B_z}{\partial z}$$

$$B_z(x) = B_0 + x \cdot G_x$$

$$B_z(y) = B_0 + y \cdot G_y$$

$$B_z(z) = B_0 + z \cdot G_z$$

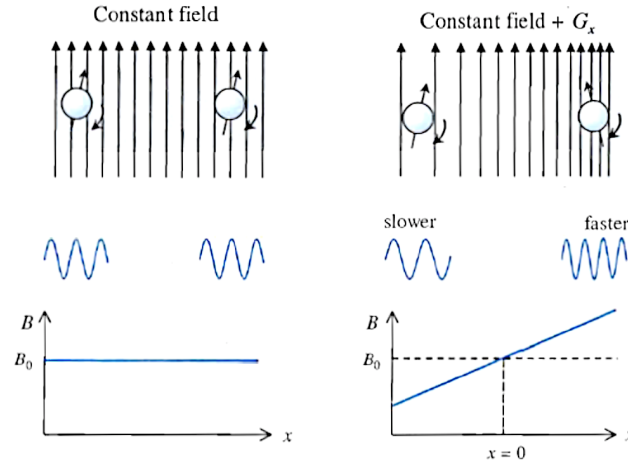
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## Magnetic Field Gradients: Freq. of Precession

What happens to the MR signal if we apply a gradient field?



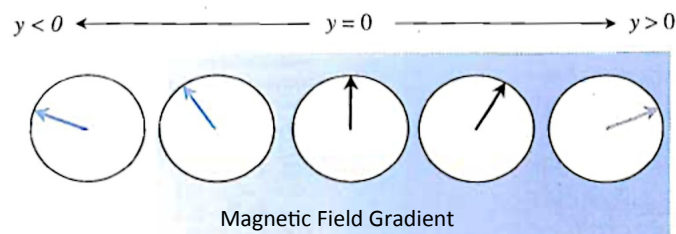
- Variation in frequency of the MR signal based on the position of spins with respect to the gradient field



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## Magnetic Field Gradients: Spin Phase

- What happens to the MR signal if we apply a gradient field for a period of time (and then turn it off)?

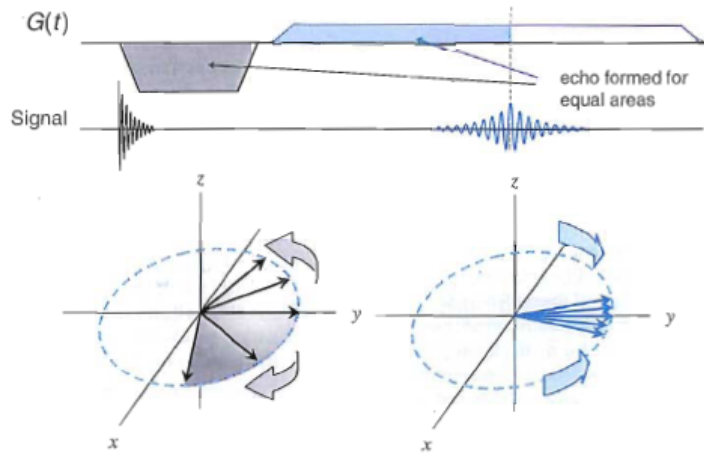


- Dephasing of spins  $\rightarrow$  Fanning out
- Angle of dephasing depends on the strength of the gradient and its duration



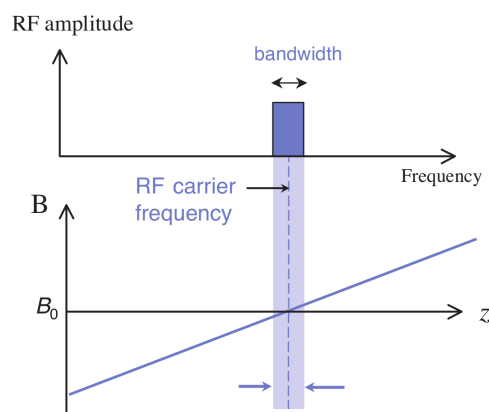
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### Dephasing and Rephasing: Gradient Echo



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### Slice Selection: Selective Excitation



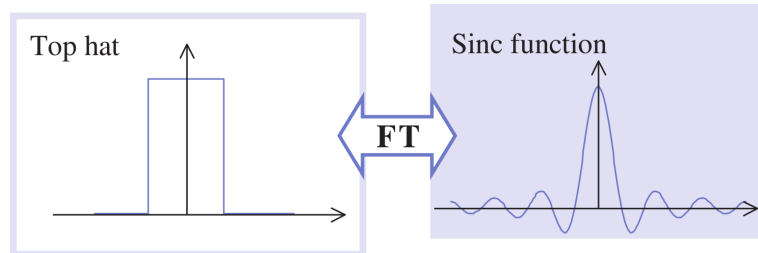
- Applying a gradient in z-direction during the RF pulse
- Larmor frequency:  $\omega_0(z) = \gamma(B_0 + G_{ss}z)$



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## Slice Selection: Selective Excitation

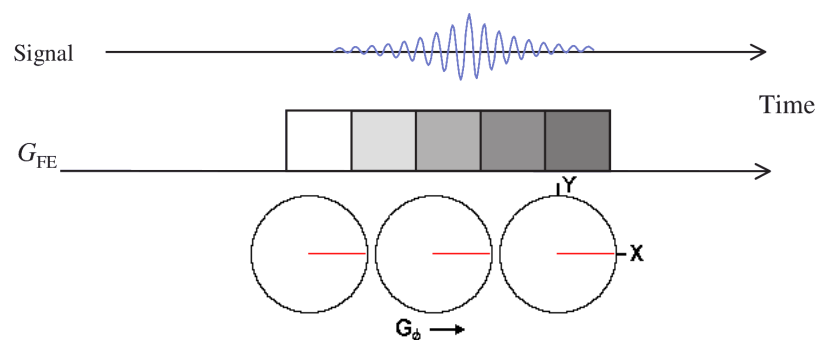
- Applying RF pulse with desired spectrum results in transversal magnetization in selected slice
- Slices with sharp transitions are desired (distinct slices)
- Envelope function of RF pulse approximates the sinc function



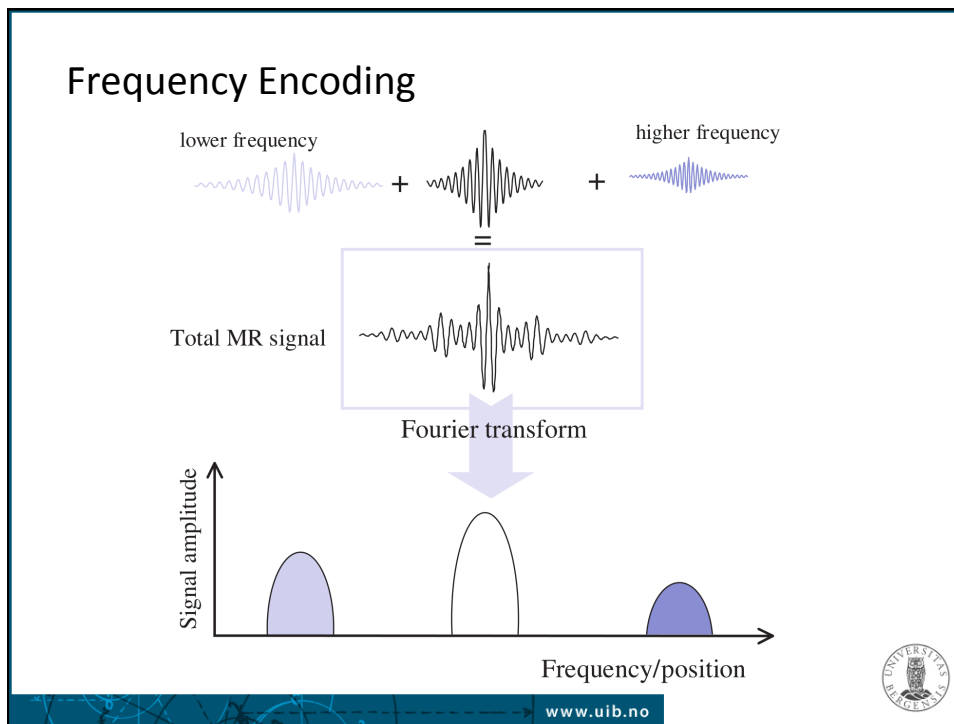
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## Frequency Encoding

- Applying gradient (e.g. along x direction) during readout
- Larmor frequency:  $\omega_0(x) = \gamma(B_0 + G_{FE}x)$
- Amplitude of measured signal is 1D projection of the object along frequency-encoding direction



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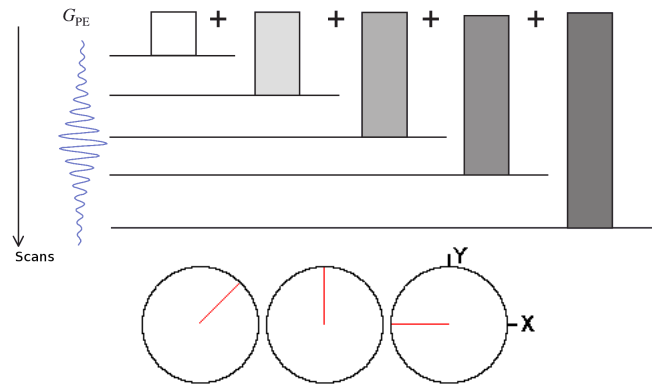
### Phase Encoding

We now encoded the signal along x (readout) direction. Can we do the same for the y direction?

- No, the frequency encoding method works only in 1 spatial dimension
- Combining gradients (x and y, for instance) would lead to an oblique slice in the frequency-encoding direction
- Another mechanism must be employed to encode the second dimension → **Phase encoding**

### Phase Encoding

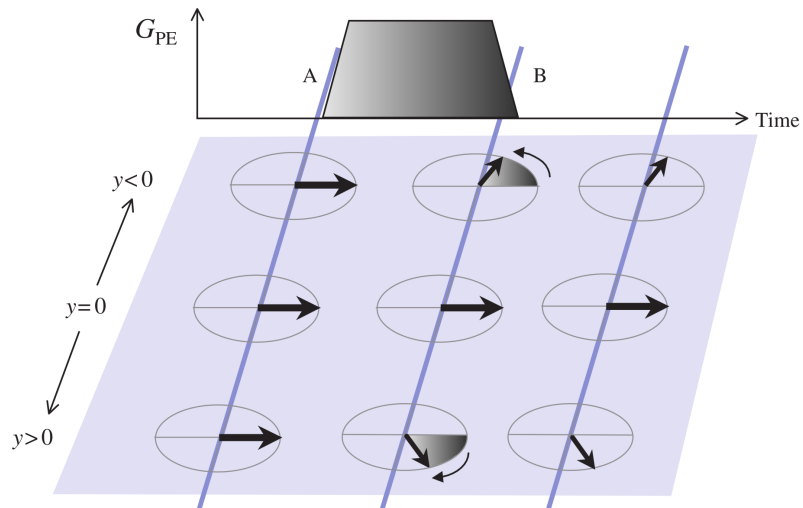
- Gradient (e.g. along y direction) is applied for a fixed amount of time before readout
- Results in a phase angle  $\phi_P = \gamma G_{PE} y T_y$



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### Phase Encoding

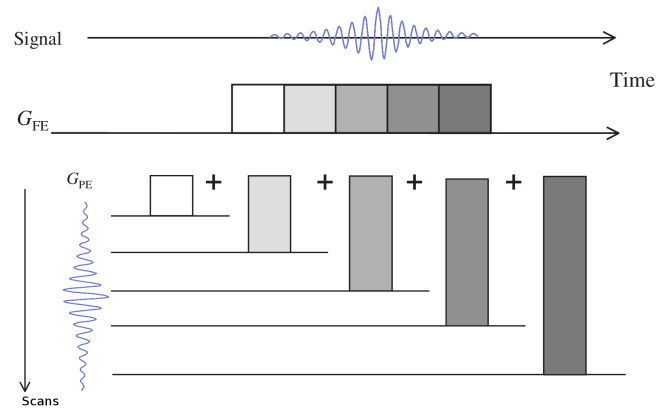


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## Phase and Frequency Encoding

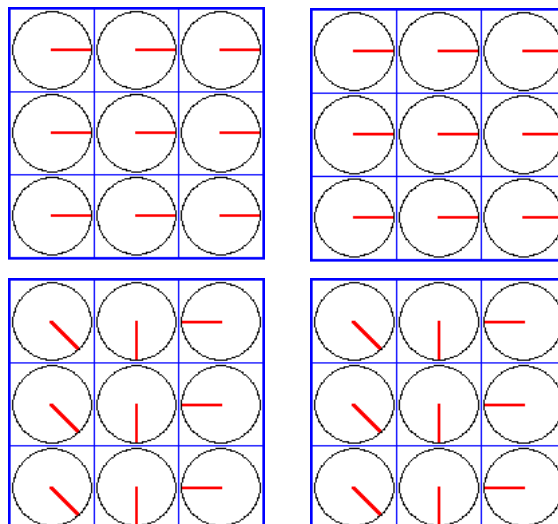
- Unlike frequency encoding, phase encoding can be done in multiple dimensions
- As many measurements as scanned rows needed
- Can think of it as frequency encoding in pseudo time



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## Phase Encoding



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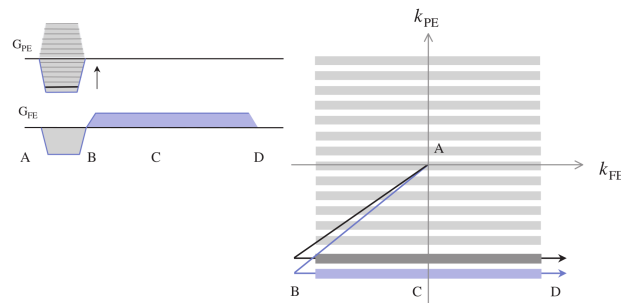


## Stepping through K-Space

- To reconstruct an image, k-space has to be adequately sampled
- Different sampling techniques
  - Minimization of imaging artifacts
  - Time-saving techniques
- Steps through k-space:

$$\Delta k_x = \gamma G_{FE} \Delta t$$

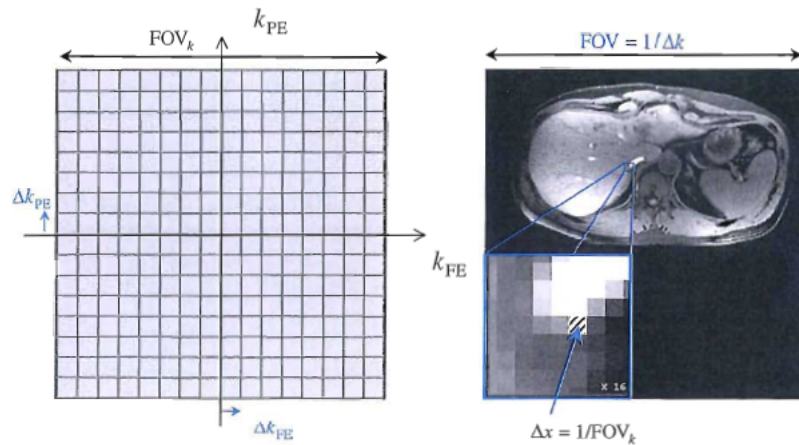
$$\Delta k_y = \gamma \Delta G_{PE} T_y$$



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## Relationship between k-space and image resolution and FOV



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## Video 2: Introduction to k-space

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### Introductory NMR and MRI with Paul Callaghan

Video 9  
Part I: Introduction to k-space



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## References

- D. W. McRobbie, E. A. Moore, M. J. Graves, M. R. Prince: MRI: From Picture to Proton, 2nd edition, Cambridge University Press, Cambridge, 2006.
- Joseph P. Hornak: The Basics of MRI:  
<http://www.cis.rit.edu/htbooks/mri/inside.htm>
- [http://epileptologie-bonn.de/cms/upload/homepage/lehnertz/JKoch\\_MRISignaltolmage.pdf](http://epileptologie-bonn.de/cms/upload/homepage/lehnertz/JKoch_MRISignaltolmage.pdf)



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