SIGNAL-TO-NOISE RATIO (SNR) IN MRI

Computing SNR

- For magnitude images (most commonly used in MRI), the signal-to-noise ratio is:

$$SNR = \frac{\text{signal amplitue}}{\text{standard deviation of noise}}$$

$$SNR = \frac{0.655 \cdot S}{\sigma_{air}} \quad \text{for magnitude data, where } 0.655 = \sqrt{\frac{4-\pi}{2}}$$

- The factor of 0.655 arises because magnitude images have a Rician noise distribution which has no negative values (unlike Gaussian noise for complex signals)

Factors Influencing SNR in MRI

- Physical and instrumental parameters
  - Magnetic field strength, B_0
  - Design of the RF coil
  - Proton density, ρ_0
  - Noise figure of the receiver pre-amplifier
  - Conductivity of the coil and sample
- Imaging sequence parameters (for 2D acquisitions)
  - Pixel size, $\Delta x \Delta y$
  - Slice thickness, Δz
  - Number of averages, N_{avg} or *NEX*
  - Readout time, T_{read}
  - Number of phase encoding steps, N_y

www.uib.no

SNR: Magnetic Field Strength

- **Signal in MRI**: $S_{MRI} \propto B_0^2$
- **Noise in MRI** comes from different sources:
  - Noise associated with resistance of coil $\propto B_0^{1/2}$
  - Noise from the body $\propto B_0^2$

$$SNR(B_0) = \frac{S_{MRI}}{\sqrt{\sigma_{coil}^2 + \sigma_{body}^2}} = \frac{B_0^2}{\sqrt{\alpha B_0^{1/2} + \beta B_0^2}}$$

- For body-noise dominance (high B_0):

$$SNR \propto \frac{B_0^2}{\sqrt{B_0^2}} = B_0$$

⇨ Reason why NMR/MRI is being performed at higher and higher fields

- For coil-noise dominance (low B_0):

$$SNR \propto \frac{B_0^2}{\sqrt{B_0^{1/2}}} = B_0^{7/4}$$

www.uib.no

SNR: RF Coil Design

- The receiver coil geometry greatly affects SNR.
- There are several types of RF receiver coils used today in MRI (in order of increasing SNR):
  - **Volume coils (saddle, birdcage)**
    - ✓ Best for imaging organs deep in the body
    - ✓ Linear[1] and quadrature[2] configurations
    - ✓ Can be used as transmit and receive coils
  - **Surface coils**
    - ✓ SNR is improved because the coil is small and is placed very close to the object being imaged
    - ✓ Best for imaging organs close to the surface of the body and brain
    - ✓ Linear and quadrature configurations

[1]what we currently have at Vivarium
[2]quadrature gives a factor of √2 better SNR than linear coil because 2 coils are used for signal detection

SNR: RF Coil Design, Cont.

- **Phased-array coils**
  - ✓ SNR is maximized because *n* very small elements are used for signal detection → parallel imaging
  - ✓ Reconstruction of signal requires special algorithms

- To maximize SNR:
  - Choose (if possible) the coil that is most suited for your application
    - ✓ E.g., brain surface coil, cardiac coil, whole body volume coil, etc.
  - The volume of the tissue imaged must optimally fill the sensitive region of the coil
  - If using a surface coil, position it as close to the interest area as possible and in the plane perpendicular to B_0

SNR: Proton Density

- <u>MRI signal is proportional to the number of protons per unit volume:</u>

$$S_{MRI} = \int_{body} M(r,t)\,dV$$

Ignore for now

$$S_{MRI} = \iiint M_0(x,y,z)e^{-i\omega_0 t}e^{-t/T_2}\left(1-e^{-t/T_1}\right)f\left(G(t)\right)dxdydz$$

- Where M_0 is the magnetization density along B_0:

$$M_0 \propto \rho_0 B_0$$

- ρ_0 is the spin (proton) density
  - in general, ρ_0 is a function of position (x,y,z) along the sample

$$\Rightarrow SNR \propto \rho_0$$

www.uib.no

SNR: Proton Density

- **<u>Demonstration:</u>**
  - Image a phantom of water and oil using a proton-density weighted sequence (with long TR and short TE times)
  - Measure *SNR* in ROI of both water and oil
  - Explain: different number of protons per unit volume in water and oil

www.uib.no

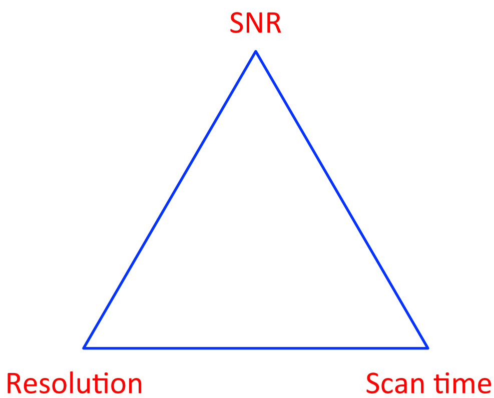
Section Summary: Dependence of SNR on Magnetic Field Strength and Spin Proton Density

$$SNR \propto B_0 \rho_0$$

=> *SNR* is proportional to the field strength

=> *SNR* is proportional to the density of protons in tissue

Motivation for what is to come:



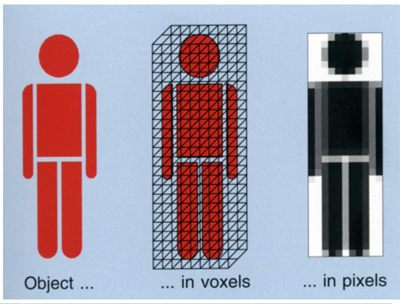
SNR: Pixel/Voxel Size

- **First, some definitions:**
  - **FIELD-OF-VIEW**:

    Is the linear extent of the imaged object: FOV_x, FOV_y (FOV_z)
  - **SPATIAL RESOLUTION**:

    Is the size of the pixels (2D) or voxels (3D) in the image: Δx, Δy, (Δz)
  - **MATRIX**:

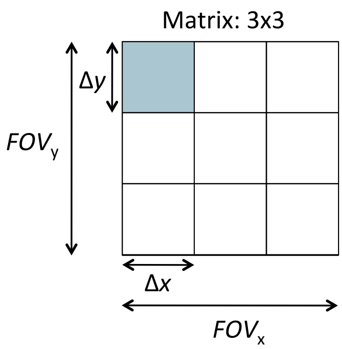
    Is the number of frequency and phase-encoding steps: N_x x N_y x (N_y)



FOV, Resolution and Matrix

- Field-of-view, resolution and matrix size are related through:

$$FOV_x = N_x \Delta x$$
$$FOV_y = N_y \Delta y$$



=> Increasing/decreasing the FOV while keeping the matrix the same will reduce/increase resolution

=> Increasing/decreasing the matrix size while keeping FOV the same will increase/reduce resolution

SNR: Pixel/Voxel Size, Cont.

- Back to MRI signal equation:

$$S_{MRI} = \int M(r,t)dV = \int M(x,y,z,t)dxdydz$$
$$S_{MRI} \propto (\Delta x)(\Delta y)(\Delta z)$$

- MRI signal is proportional to the size of the unit volume being imaged:
  - = pixel ($\Delta x \Delta y$) in 2D imaging
  - = voxel ($\Delta x \Delta y \Delta z$) in 3D imaging

  A larger voxel will contain more spins than a small voxel

$$SNR \propto \Delta V$$

www.uib.no

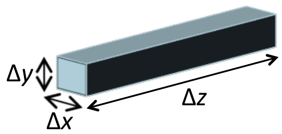
SNR: Pixel/Voxel Size, Cont.

- **Demonstration:**
  - Collect an image of water phantom:
    - $FOV_x = FOV_y$ = 4.0 cm; Matrix = 256 x256 => $\Delta x_1, \Delta y_1$
    - $FOV_x = FOV_y$ = 8.0 cm; Matrix = 256 x256 => $\Delta x_2, \Delta y_2 = 2\Delta x_1, \Delta y_1$
    - $FOV_x = FOV_y$ = 4.0 cm; Matrix = 128 x128 => $\Delta x_3, \Delta y_3 = 2\Delta x_1, \Delta y_1$
  - Measure *SNR* in an ROI each case and compare
  - Explain
    - $SNR_2 = 4SNR_1$ (resolution decreased by 4, matrix was unchanged)
    - $SNR_3 = 2SNR_1$ (resolution decreased by 4, **BUT**: matrix changed, see further)

www.uib.no

SNR: Slice Thickness (2D Imaging)

- In 2D imaging we collect slices of thickness Δz and pixel size $\Delta x \Delta y$.



- In general,
  - $\Delta z >> \Delta x \Delta y$

    i.e, in plane resolution is much bigger than slice thickness

    => **Partial volume effect:** a single voxel contains a mixture of multiple tissue values

    => partial volume effect is reduced by increasing imaging resolution
- Similarly as for a voxel, the MRI signal and therefore SNR is proportional to the thickness of the slice:

$$SNR \propto \Delta z$$

SNR: Slice Thickness (2D Imaging)

- **Demonstration:**
  - Collect an image of water phantom:
    - Slice thickness Δz_1
    - Slice thickness $\Delta z_2 = 2\Delta z_1$
  - Measure *SNR* in an ROI in each case and compare
  - Explain
    - $SNR_2 = 2SNR_1$ (slice thickness increased by factor 2)

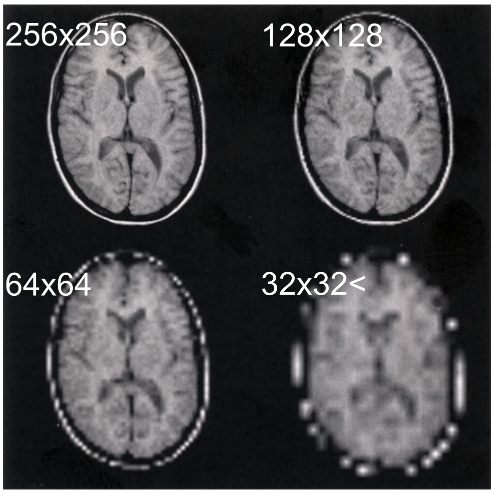
SNR: Summary of Spin Density/Resolution Effects

$$SNR \propto \rho_0 \Delta x \Delta y \Delta z = \rho_0 \Delta V$$

=> *SNR* is proportional to the total number of spins in a unit volume

Resolution in Practice

- Dependent on what we are about to investigate, we decide upon a matrix size.
- Higher resolution (512X512) may give better detail about fine structures, but the noise also increases.
- When resolution is too low, the images get "blurred". This is due to the partial volume effect.
- We commonly use 256x256 for imaging, and 128x128 for diffusion experiments.



- What can you do if you need a certain resolution to resolve the structures you are imaging but your data is too noisy?

SNR: Number of Averages

- One can improve SNR by averaging separate measurements of the same *k*-space region (i.e., each line of *k*-space is collected several times)
- Adding two measurement together =>
  - MRI signal amplitudes add (because signal occurs at the same place each time it is collected)
  - MRI noise variances also add (when noise is random and uncorrelated for each measurement) =>

$$SNR = \frac{S_{MRI,1} + S_{MRI,2}}{\sqrt{\sigma_1^2 + \sigma_2^2}} = \frac{2S}{\sqrt{2\sigma^2}} = \sqrt{2}\frac{S_{MRI}}{\sigma}$$

$$SNR \propto \sqrt{N_{avg}}$$

=> E.g., to double SNR, the number of averages and therefore scan time has to be increased by a factor of 4.

SNR: Number of Averages

- **Demonstration:**
  - Collect an image of water phantom:
    - Number of averages $N_{avg,1}$
    - Number of averages $N_{avg,2}=4N_{avg,1}$
  - Measure *SNR* in an ROI in each case and compare
  - Explain
    - $SNR_2=2SNR_1$ (number of averages increased by factor 4)

SNR: Readout Time (Receiver Bandwidth)

- The readout time is defined as $T_{read}=N_x\Delta t$, where
  - N_x is the number of steps along the frequency/readout direction
  - Δt is the sampling interval and is related to the signal bandwidth (range of frequencies sampled during the readout): $BW=1/\Delta t$

- If T_{read} is doubled by doubling Δt while keeping N_x the same:
  - MRI signal amplitude is unchanged (independent of *BW*)
  - MRI noise variance is halved (noise variance is proportional to *BW* since noise occurs at all frequencies and randomly in time)

$$SNR = \frac{S_{MRI}}{\sqrt{\sigma^2/2}} = \sqrt{2}\,\frac{S_{MRI}}{\sigma} \Rightarrow$$

Doubling T_{read} increased *SNR* by a factor of √2.

$$SNR \propto \sqrt{T_{read}}$$

SNR: Readout Time (Receiver Bandwidth)

- **Demonstration:**
  - Collect an image of water phantom:
    - Receiver bandwidth BW_1
    - Receiver bandwidth $BW_2=2BW_1$, leave matrix unchanged
  - Measure *SNR* in an ROI in each case and compare
  - Explain
    - $SNR_2=SNR_1/\sqrt{2}$ (Δt and therefore T_{read} decreased by factor of 2)

www.uib.no

SNR: Summary of Acquisition Time Effects

$$SNR \propto \sqrt{N_{avg} T_{read}}$$
$$SNR \propto \sqrt{\text{measurement time}}$$

=> *SNR* is proportional to the square root of the cumulative scan time

www.uib.no

SNR: Number of Phase Encoding Steps

- Number of phase encoding steps, N_y and N_z (in 3D imaging) effects the total scan time:

$$T_{scan} \propto N_y N_z$$

- It follows, that *SNR* in MRI is:

$$SNR \propto \sqrt{N_y N_z}$$

www.uib.no

SNR: Number of Phase Encoding Steps

- **Demonstration:**
  - Collect an image of water phantom:
    - Number of phase encoding steps $N_{y,1}$=256 at $FOV_{y,1}$
    - Number of phase encoding steps $N_{y,2}=2N_{y,1}$=512 at $FOV_{y,2}=2FOV_{y,1}$
  - Measure *SNR* in an ROI in each case and compare
  - Explain
    - $SNR_2=\sqrt{2}SNR_1$ (N_y increased by factor of 2, resolution was unchanged)

www.uib.no

Section Summary: Dependence of SNR on Acquisition Parameters

$$SNR \propto \Delta x \Delta y \Delta z \sqrt{N_{avg} N_x N_y N_z \Delta t}$$

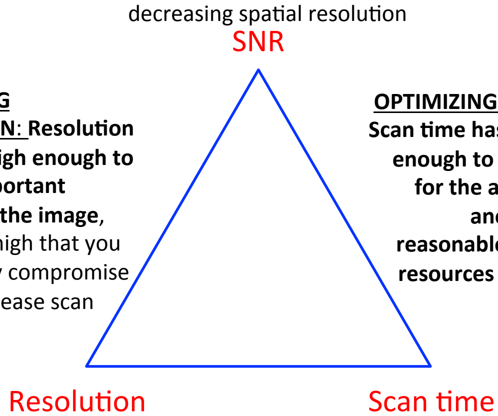
$$SNR \propto \Delta x \Delta y \Delta z \sqrt{N_{avg} N_y N_z T_{read}}$$

=> *SNR* is proportional to voxel volume (=> resolution) and sqrt of the scan time

$$SNR \propto \text{3D Imaging resolution} \sqrt{\text{scanning time}}$$

Interplay Between SNR, Resolution and Scan Time

OPTIMIZING SNR:
SNR has to be high enough for a reliable analysis of data.
Can increase SNR by either increasing scan time or decreasing spatial resolution



OPTIMIZING RESOLUTION: Resolution has to be high enough to resolve important features in the image, but not so high that you significantly compromise SNR or increase scan time

OPTIMIZING SCAN TIME: Scan time has to be short enough to be tolerable for the animal under anesthesia and reasonable in terms of resources used for the experiment

Section Summary: Dependence of SNR on Acquisition Parameters

- **Demonstration:**
  - Collect an image of water phantom:
    - $FOV_x = FOV_y$ = 4.0 cm; Matrix = 256 x256
    - $FOV_x = FOV_y$ = 4.0 cm; Matrix = 128 x128
  - Measure *SNR* in an ROI in each case and compare
  - Explain
    - SNR_2=2SNR_1, resolution decreased by 4, also matrix changed, so:

$$\Delta x_2 = 2\Delta x_1$$
$$\Delta y_2 = 2\Delta y_1$$
$$N_{x,2} = N_{x,1}/2$$
$$N_{y,2} = N_{y,1}/2$$
$$SNR_2 = 2 \cdot 2 \cdot \sqrt{1 \cdot \frac{1}{2} \cdot \frac{1}{2}} SNR_1 = \frac{4}{2} SNR_1 = 2SNR_1$$

www.uib.no

Section Summary: Dependence of SNR on Acquisition Parameters

- **Demonstration:**
  - Collect an image of water phantom:
    - $FOV_x = FOV_y$ = 4.0 cm; Matrix = 256 x256, $N_{avg,1}$=4
    - $FOV_x = FOV_y$ = 4.0 cm; Matrix = 128 x128, $N_{avg,2}$=1
  - Measure *SNR* in an ROI in each case and compare
  - Explain
    - SNR_2=1SNR_1

$$\Delta x_2 = 2\Delta x_1$$
$$\Delta y_2 = 2\Delta y_1$$
$$N_{x,2} = N_{x,1}/2$$
$$N_{y,2} = N_{y,1}/2$$
$$N_{avg,2} = \frac{1}{4} N_{avg,1}$$
$$SNR_2 = 2 \cdot 2 \cdot \sqrt{\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2}} SNR_1 = \frac{4}{4} SNR_1 = SNR_1$$

www.uib.no